

Numerical Analysis of Ordinary Differential Equations Exercises

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Exercise sheet 4

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Exercise 4.1 (Local error control of ERK methods)

In Algorithm 2.4.2 of the lecture notes the optimal step size is determined by

$$h_{opt} = h_k \sqrt[p+1]{\frac{\varepsilon}{\|y_k - \hat{y}_k\|}}$$

where ε is a tolerance given by the user, $y_k - \hat{y}_k$ is an estimate of the local error for the current step and $\|\cdot\|$ an appropriate norm.

- a) If the local error estimate is greater than the tolerance ε , i.e.

$$\|y_k - \hat{y}_k\| > \varepsilon,$$

y_k is rejected and step k is repeated with step size $h_k^{new} = h_{opt}$. Why is that a good choice? Give reasons.

- b) If the step is accepted, i.e.

$$\|y_k - \hat{y}_k\| \leq \varepsilon,$$

the step size for the next step is set to $h_{k+1} = h_{opt}$. Why is that a good choice? Give reasons.

Exercise 4.2 (Scaled norm)

Consider

$$\|v\|_s := \left(\frac{1}{d} \sum_{i=1}^d \left(\frac{v_i}{s_i} \right)^2 \right)^{\frac{1}{2}}$$

where $s_i > 0$.

- a) Show that $\|\cdot\|_s$ is a norm.
b) In time step control algorithm s might be chosen as

$$s_i = \max \{s_{\min}, |(y_{k-1})_i|\}.$$

What is the difference between $\|\cdot\|_s$ and the Euclidean norm when measuring errors of y_k , for example the local error of y_k .