Numerical Analysis of Ordinary Differential Equations
Exercises

Summer semester 2016

Universität Heidelberg - IWR
Prof. Dr. Guido Kanschat, Dr. Dörte Beigel,
Stefan Meggendorfer, Philipp Siehr

Exercise sheet 7
Until: 09.06.2016

Exercise 7.1 (**Implicit Runge-Kutta methods**)
Consider a 2-stage IRK method of the form

$$\begin{array}{c|cc} u & v & u-v \\ 1-u & 1-u-w & w \\ \hline & 1/2 & 1/2 \end{array}$$

a) Show that all these methods are at least of order 2.

b) Which conditions have to be satisfied such that they are of order 3?

c) Construct a method of order 4.

Exercise 7.2 (**BDF methods with variable stepsizes**)
BDF methods seek a new approximation y_k such that the interpolation polynomial

$$y(t) = \sum_{r=0}^{s} L_{s-r}(t) y_{k-r}$$

through past approximations $y_{k-s}, \ldots, y_{k-1}$ and y_k satisfies the ODE in t_k:

$$y'(t_k) = f(t_k, y_k).$$

$L_{s-r}(t)$ are the fundamental Lagrange polynomials of order s (see e.g. Wikipedia).

Consider a BDF method with $s = 2$ and three different stepsizes $h_1 = t_k - t_{k-1}$ and $h_2 = t_{k-1} - t_{k-2}$.

a) Derive expressions for the coefficients α in

$$\sum_{r=0}^{s} \alpha_{s-r} y_{k-r} = h f(t_k, y_k).$$

b) Interpret the coefficients given in the lecture.

Exercise 7.3 (**Consistency order and stability of LMM**)
Consider the explicit multistep method given by

$$y_k + \alpha_1 y_{k-1} + \alpha_0 y_{k-2} = h(\beta_1 f(t_{k-1}, y_{k-1}) + \beta_0 f(t_{k-2}, y_{k-2})).$$

a) Determine α_0, β_1, β_0 depending on α_1 such that the LMM has at least consistency order 2.

b) For which values of α_1 is the method stable?

c) Which method is obtained when $\alpha_0 = 0$ and $\alpha_1 = -1$?

d) Can α_1 be chosen such that the method has convergence order 3?