Numerical Analysis of Ordinary Differential Equations
Exercises

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Exercise sheet 11
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Consider for $k \in \mathbb{R}, k \neq 0$ the differential equation

$$u'' = k^2 u, \tag{11.1}$$

with the two linearly independent solutions $e^{k(t-t_0)}$ and $e^{-k(t-t_0)}$.

Exercise 11.1 (**Solvability of BVP**)

a) Transform this equation into a first order system for the vector $(u, v)^T$ and determine the fundamental matrix at t for the initial value $\mathbb{I}$ at t_0.

   Note that $\sinh z = \frac{e^z - e^{-z}}{2}$ and $\cosh z = \frac{e^z + e^{-z}}{2}$.

b) On the interval $[a, b]$ consider the boundary value problem that consists of (11.1) and the boundary conditions $u(a) = g_a$ and $u(b) = g_b$. Rewrite this BVP in terms of the first order system with boundary conditions of the form

$$B_a \begin{pmatrix} u(a) \\ v(a) \end{pmatrix} + B_b \begin{pmatrix} u(b) \\ v(b) \end{pmatrix} = \begin{pmatrix} g_a \\ g_b \end{pmatrix},$$

   i.e. determine B_a and B_b.

c) Use Corollary 5.3.13 to determine whether this system has a unique solution for $a = -1$, $b = 1$. If so, determine the solution for $g_a = g_b = 1$.

   Hint: Consider $\cosh(kt)$.

Exercise 11.2 (**Single Shooting for BVP**)
Perform one step of the single shooting method for this BVP with $a = -1, b = 1$ and $g_a = g_b = 1$ (use the BVP in the first order form of Exercise 11.1 b).