IWR – Universität Heidelberg
Prof. Dr. Guido Kanschat
Stefan Meggendorfer

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Homework No. 2
Numerical Methods for PDE, Winter 2016/17

Problem 2.1: Given the sequence of functions

$$f_n(x) = \frac{|x|^3}{|x^2| + \frac{1}{n}}.$$

(a) Show that f_n is continuously differentiable.

(b) Show that $f_n \to |x|$ in $H^1(-1,1)$ as $n \to \infty$.

Hint: Use de l'Hôpital's rule for quotients of sequences which converge to infinity.

Problem 2.2: Let $\Omega = (-1,1)$. Show that on the space of continuous functions on Ω the norms

$$\|f\|_\infty = \sup_{x\in\Omega} |f(x)| \quad \text{and} \quad \|f\|_2 = \left(\int_\Omega |f(x)|^2 \,\mathrm{d}x\right)^{\frac{1}{2}}$$

are not equivalent.

Hint: Find a sequence which is bounded in one norm and tends to zero with respect to the other.

Problem 2.3: Friedrichs' inequality

(a) Prove Friedrichs' inequality

$$\|u\|_{L^2(\Omega)} \le c\|u'\|_{L^2(\Omega)}, \qquad \text{with } c = b - a$$

for $\Omega = (a,b)$ and functions $u \in C_0^1(\Omega)$.

(b) Generalize the proof for functions in $H_0^1(\Omega)$, using that each function in $H_0^1(\Omega)$ is the limit of a sequence in $C_0^1(\Omega)$.

Problem 2.4: Weak formulation of Robin boundary value problem

Given is the following Robin-boundary problem

$$\begin{aligned} -\Delta u(x) &= f(x), && \text{in } \Omega, \\ \partial_n u(x) + \mu u(x) &= g(x), && \text{on } \partial\Omega, \end{aligned}$$

with a bounded domain $\Omega \subset \mathbb{R}^n$, which has a smooth boundary $\partial\Omega$ and $\mu > 0$.

(a) Formulate the problem weakly for functions $u \in H^1(\Omega)$.

(b) Equip $H^1(\Omega)$ with an inner product and a norm, such that you can prove existence and uniqueness of a solution to your weak formulation by Riesz representation theorem. Bonus points for showing that the inner product is indeed one.

(c) Set $\mu = 0$. Is there still a unique solution?