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Homework No. 7
Numerical Methods for PDE, Winter 2016/17

Problem 7.1: Recapitulate Galerkin orthogonality and Céa's lemma.

Problem 7.2: Trace inequality for polynomials Let K be a square of diameter h and ∂K its boundary. Show that for any polynomial $p \in P_k$ holds

$$\|p\|^2_{L^2(\partial K)} \leq c\left(h\|\nabla p\|^2_{L^2(K)} + \frac{1}{h}\|p\|^2_{L^2(K)}\right)$$

with a constant $c > 0$.

Problem 7.3: Error estimate for the mean value Let $\Omega \subset \mathbb{R}^2$ be a convex polygonal domain. For the homogenious Laplace-equation in weak form

$$\text{Find } u \in H^1_0, \text{ s.t.} \qquad \int_\Omega \nabla u \cdot \nabla \varphi \, dx = \int_\Omega f\varphi \, dx \quad \forall \varphi \in H^1_0(\Omega),$$

we have the dual problem

$$\text{Find } z \in H^1_0, \text{ s.t.} \qquad \int_\Omega \nabla v \cdot \nabla z \, dx = J(v) \quad \forall v \in H^1_0(\Omega),$$

where $J(\cdot)$ is the so called "error functional". Let $u_h \in V_h \subset V$ be an approximation of u by a FEM of order $k \geq 1$ and let $e_h = u - u_h$ denote the error. Derive the error estimate

$$|\int_\Omega e_h dx| \leq ch^{2k}\|f\|_{H^{k-1}(\Omega)},$$

with a constant $c > 0$, by having at hand the following regularity estimates for $s \geq 0$ (assuming that $f \in H^s(\Omega)$)

$$\|u\|_{H^{s+2}(\Omega)} \leq c\|f\|_{H^s(\Omega)}, \qquad \|z\|_{H^{s+2}(\Omega)} \leq c.$$