IWR – Universität Heidelberg
Prof. Dr. Guido Kanschat
Stefan Meggendorfer

Due date: **22.12.2016**

Homework No. 9
Numerical Methods for PDE, Winter 2016/17

Problem 9.1: Special A Posteriori Error Estimators

Consider the general Neumann problem

$$-\nabla \cdot (\alpha(x)\nabla u(x)) + \gamma(x)u(x) = f(x) \qquad \text{in } \Omega$$
$$n(x) \cdot (\alpha(x)\nabla u(x)) = g(x) \qquad \text{on } \partial\Omega$$

with smooth data functions α, γ, f, g and $\alpha(x) \geq \tilde{\alpha} > 0$ and $\gamma(x) \geq \tilde{\gamma} > 0$. The domain $\Omega \subset \mathbb{R}^2$ is again a convex polygon.

(a) Derive the weak formulation as usual and discretize the equation with piecewise linear finite elements.

(b) Discuss unique existence of solutions u and u_h.

(c) Formulate the dual problem for a general error functional $J(\cdot)$.

(d) Derive an error representation for the error $e_h = u - u_h$ in the functional $J(\cdot)$ by cellwise dual weighted residuals analogously to the proceeding in the lecture.

(e) Derive an a posteriori error estimation for the error in the L^2-norm $\|e_h\|_{L^2(\Omega)}$.

Hint: Use the nodal interpolant $\psi_h := \Pi_{\mathcal{Z}} z$ and their properties.