IWR – Universität Heidelberg
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Due date: **Discussion in first tutorials in January**

Homework No. 10
Numerical Methods for PDE, Winter 2016/17

Problem 10.1: Short questions

(a) Consider a function $u \in L^2(\Omega)$. How can you formulate the weak Laplacian for this function?

(b) Formulate the Riesz representation theorem and the Lax-Milgram lemma.

(c) In which sense is the Lax-Milgram lemma an enhancement of the Riesz representation theorem?

(d) What does V-elliptic mean for a bilinear form $a(u, v)$ with u, v in V?

(e) Is the bilinear form $a(u, v) = (\nabla u, \nabla v)$ V-elliptic for $V = H^1(\Omega)$?

(f) Explain what is meant by the term 'Galerkin orthogonality'.

(g) Formulate Ceá's lemma and state the requirements for this result.

(h) Formulate a linear error functional $J(\varphi)$ which represents the L^2-Norm for $\varphi = u - u_h$.

(i) Describe the duality argument (Aubin-Nitsche trick) for error estiamtes in 'weak' norms. What is it used for?

(j) Describe the concept of parametric finite elements.

(k) How do node values induce continuity into a finite element space?

(l) What does the term unisolvence mean for a polynomial ansatz space?

(m) When is a triangulation called shape regular?

(n) Write a typical a priori error estimate.

(o) State the Bramble-Hilbert Lemma.

(p) Describe the connection between error estimates, transformation of the reference cell and the Bramble-Hilbert Lemma.

(q) Which order of convergence can we obtain for the L^2- and the H^1-norm of an approximation with quadratic finite elements in the best case?

(r) What is error pollution?

(s) What is the difference between a-priori and a-posteriori error estimates?

(t) Describe the concept of a dual weighted error estimator.