1.1.11 Problem: Given the vector space of square matrices $X = \mathbb{R}^{d\times d}$ with the Frobenius inner product

$$\langle A, B\rangle = A : B = \sum_{ij} a_{ij} b_{ij}. \tag{1}$$

Show that the subspaces of symmetric and skew-symmetric matrices, respectively, are orthogonal to each other and X is the direct sum of those.

1.1.16 Problem: Let the space $V = H_0^1(\Omega; \mathbb{R}^d)$ be equipped with the inner product $\langle u, v\rangle = a(u, v)$ with the bilinear form of the Lamé-Navier equations and the corresponding norm $\|.\|_V$.

Show using techniques from the standard theory of elliptic partial differential equations:

1. The weak formulation has a unique solution for which there holds
$$\|u\|_V \le \|f\|_{V^*}.$$
2. The "energy estimate" for conforming finite element approximation with a space $V_h \subset V$
$$\|u - u_h\|_V = \inf_{v_h \in V_h} \|u - v_h\|_V.$$
3. The H^1-error estimate
$$\|u - u_h\|_{H^1} \le \frac{2\mu + d\lambda}{2c_K\mu} \inf_{v_h\in V_h} \|u - v_h\|_{H^1}. \tag{2}$$
Use the fact that the space V can be composed into the space V^0 of divergence-free functions ($\nabla\cdot v = 0$) and its complement.
4. For $\lambda \gg \mu$, the previous estimate is useless. Can it be improved easily? In view of the "energy estimate", can you think of conditions?