2.3.6 Problem: Show that Theorem 2.3.3 can be extended to the case with right hand side $f(v) + g(q)$ with $g \in Q^*$.

2.4.10 Problem: Let the following interpolation estimates hold:

$$\inf_{v_h \in V_h} \|u - v_h\|_V = \mathcal{O}(h^k), \qquad \inf_{q_h \in Q_h} \|p - q_h\|_V = \mathcal{O}(h^k).$$

Then, the estimates in Theorem 2.4.7 are asymptotically optimal if and only if there are constants $\tilde{\gamma} > 0$ and $\tilde{\beta} > 0$ independent of h such that

$$\gamma_h \geq \tilde{\gamma}, \quad \beta_h \geq \tilde{\beta}, \tag{3}$$

independent of h.

2.4.13 Problem: Let $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{k \times n}$, $k \leq n$. Moreover, assume that B has full rank and that A is symmetric and positive definite.
Consider the problem

$$\begin{pmatrix} A & B^* \\ B & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} F \\ G \end{pmatrix} \tag{*}$$

1. Prove that then $S := BA^{-1}B^*$ is symmetric and positive definite, too. How can this matrix be used to solve (*)?
2. Show that
$$P := I - B^*(BB^*)^{-1}B.$$
is a projektor on the kernel of B with $\|P\|_2 = 1$.
3. Show for the case $G = 0$ that x is a solution of
$$PAPx = PF$$
if (x, y) is a solution of (*).