3.2.7 Problem: The domain $\Omega = [0,1]^2$ is decomposed into $N \times N$ congruent squares where each of them is again divided into two triangles. The decomposition $\mathcal{T}_h$ is given by these triangles.

We again choose piecewise linear ansatz functions for the velocity for V_h (vanishing on $\partial\Omega$) and piecewise constant ansatz functions for Q_h.

Is there a N and an orientation of the triangles such that $V_h \times Q_h$ is inf-sup stable?

3.2.8 Problem: Let $\Omega = (0,1)^2$ be the unit square and let the mesh consist of Cartesian squares of side length $1/n$. Choose $V_h \subset V$ based on bilinear shape functions. Show that the piecewise constant pressure function $p_c = \pm 1$ in a checkerboard fashion is in the kernel of B_h^T, that is

$$b(v_h, p_c) = 0 \quad \forall v_h \in V_h.$$

3.2.23 Problem: Show that the MINI element can be generalized to quadrilateral meshes. Design a bubble space b_Q of minimal tensor degree such that

$$V_h = \left(H_h^1(\mathbb{Q}_1) \oplus B_h(b_Q) \cap V\right)^2, \qquad Q_h = H_h^1(\mathbb{Q}_1) \cap Q.$$

Discuss extensions to tetrahedra and hexahedra in three dimensions.