Numerical Analysis of Ordinary Differential Equations

Exercises

Summer Semester 2018

Universität Heidelberg - IWR
Prof. Dr. Guido Kanschat, Dr. Dörte Jando,
Bastian Boll, Pablo Lucero

Exercise Sheet 1
Until: Mo, 20.04.2018, Noon (12:00h)

There will be no scores given and accumulated over the semester. Nevertheless, we strongly recommend solving the exercises. If you choose to hand in your solutions they will be annotated according to completeness and mathematical correctness. Please hand in your solution in groups of at least 2 or 3 students.
Important: Please register to one of the exercise groups in **Müsli**.

Problem 1.1 (**Modeling Oblivion**)
Suppose that a student has learned a certain amount of learning material. Unfortunately, he will forget a lot of it over time. Being optimistic, a certain percentage of the knowledge will remain for ever.

a) Make appropriate assumptions on the rate of oblivion and set up a mathematical model.

b) Solve the ODE and sketch the solution.

Problem 1.2 (**A Model to determine the time of death**)
In forensic investigations it is crucial to determine the exact time of death for a found body. Let us assume that the temperature of the found body fulfills Newton's law of heat transfer.

At midnight a body is found with temperature 29.4° C at a place where the temperature of the environment is 21.1° C. The body is immediately brought into pathology where the temperture is 4.4° C. After one hour the temperature of the body is 15.6° C. Determine the time of death.

Problem 1.3 (**Matrixvalued Mappings**)
Let $A \in \mathbb{C}^{n\times n}$. We define the exponential function by

$$e^A := \sum_{k=0}^{\infty} \frac{1}{k!} A^k = \lim_{m\to\infty} \sum_{k=0}^{m} \frac{1}{k!} A^k.$$

Assume absolute convergence.

a) Let $D \in \mathbb{C}^{n\times n}$ be a diagonal matrix. Calculate e^D.

b) Let $A \in \mathbb{C}^{n\times n}$ and $S \in \mathbb{C}^{n\times n}$ invertible. Show that $e^{SAS^{-1}} = Se^A S^{-1}$.

c) Consider the following IVP for vector-valued functions $u : [0, T] \to \mathbb{C}^n$

$$\begin{aligned} u' &= Au, \\ u(0) &= u_0. \end{aligned}$$

Assume that $A \in \mathbb{C}^{n \times n}$ is diagonalisable. Show that $u(t) = e^{At} u_0$ is a solution. Can we expect uniqueness?

Problem 1.4 (ODEs of Higher Order)
Consider the following system:

$$\begin{aligned} v^{(vi)}(x) - a(x)u'(x) &= f(x), \\ u''(x) + b(x)v(x) &= g(x). \end{aligned}$$

Transform this system into a system consisting of ODEs with order 1.