Numerical Analysis of Ordinary Differential Equations

Exercises

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Exercise Sheet 3
Until: Mo, 07.05.2018, Noon (12:00h)

Problem 3.1 (**Uniqueness of Solutions**)

a) Consider the following initial value problem:

$$u'(t) = -u(t)^2, \qquad t \geqslant 0, \qquad u(0) = -1.$$

Find a solution. Is the solution unique in the region of existence?

b) Show that $u(t) = (\frac{3}{4}t + 1)^{\frac{4}{3}}$ is a solution of the following IVP:

$$u'(t) = u(t)^{\frac{1}{4}}, \qquad t \geqslant 0, \qquad u(0) = 1.$$

Is this solution unique?

c) Now consider the slightly changed IVP:

$$u'(t) = u(t)^{\frac{1}{4}}, \qquad t \geqslant 0, \qquad u(0) = 0.$$

Find a solution and discuss uniqueness.

Remark: You can either guess the solutions or use separation of variables.

Problem 3.2 (**Local and Global Errors**)

Consider the following IVP

$$u'(t) = -2tu(t)^2, \quad u(t_0) = u_0$$

where $t_0 \geq 1$ and $u_0 > 0$ is assumed.

a) Use separation of variables to find an analytic solution.

b) For $t_0 = 1$ and $u_0 = 1/2$ perform two steps of the explicit Euler method with $h = 1/2$.

c) Determine the local and global errors at $t_1 = 3/2$ and $t_2 = 2$. What do you observe?

d) Perform one step of the Heun method. How does the error behave compared to the approximation computed using the Euler method?

Problem 3.3 (Consistency Order of Heun Method)

Prove that the Heun method is consistent of order 2.

a) Use a standard ODE

$$u'(t) = f(t, u(t)), \quad u(0) = u_0$$

for the proof.

b) Use an autonomous ODE

$$u'(t) = f(u(t)), \quad u(0) = u_0$$

for the proof. What is the difference?