Numerical Analysis of Ordinary Differential Equations

Exercises

Summer Semester 2018

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Exercise Sheet 4
Until: Mo, 14.05.2018, Noon (12:00h)

Problem 4.1 (**Debugging**)

Assume you implemented an integration method to solve an IVP or you obtained a code where somebody else implemented an integration method to solve an IVP.

How would you check if the program works correctly? What would you do to find bugs? Describe a reasonable strategy.

Problem 4.2 (**Local error control of ERK methods**)
In Algorithm 2.4.2 of the lecture notes the optimal step size is determined by

$$h_{opt} = h_k \sqrt[p+1]{\frac{\varepsilon}{||y_k - \hat{y}_k||}}$$

where ε is a tolerance given by the user, $y_k - \hat{y}_k$ is an estimate of the local error for the current step and $|| \cdot ||$ an appropriate norm.

a) If the local error estimate is greater than the tolerance ε, i.e.

$$||y_k - \hat{y}_k|| > \varepsilon,$$

y_k is rejected and step k is repeated with step size $h_k^{new} = h_{opt}$. Why is that a good choice? Give reasons.

b) If the step is accepted, i.e.

$$||y_k - \hat{y}_k|| \leq \varepsilon,$$

the step size for the next step is set to $h_{k+1} = h_{opt}$. Why is that a good choice? Give reasons.

Problem 4.3 (**Scaled norm**)
Consider

$$||v||_s := \left(\frac{1}{d} \sum_{i=1}^{d} \left(\frac{v_i}{s_i} \right)^2 \right)^{\frac{1}{2}}$$

where $s_i > 0$.

a) Show that $|| \cdot ||_s$ is a norm.

b) In time step control algorithm s might be chosen as

$$s_i = \max \left\{ s_{\min}, |(y_{k-1})_i| \right\}.$$

What is the difference between $|| \cdot ||_s$ and the Euclidean norm when measuring errors of y_k, for example the local error of y_k.