

# Numerical Analysis of Ordinary Differential Equations

## Exercises

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Exercise Sheet 5  
Until: Tue, 22.05.2018, Noon (12:00h)

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### Problem 5.1 (Explicit vs. implicit Euler)

Consider the following IVP

$$\begin{aligned}u^{(3)}(t) + u'(t) &= tu(t), \\ u''(2) &= 2, \\ u(2) &= 0, \\ u'(2) &= 1.\end{aligned}$$

- Convert the IVP into a system of first order ODEs.
- Compute approximations to the solution of the first order system at  $t = 2.5$  by one step of the explicit and the implicit Euler method.
- What is the difference between the explicit and implicit Euler method? Give a geometric interpretation.

### Problem 5.2 (Properties of the Theta-method)

Prove the following properties of the Theta-method

$$y_1 = y_0 + h((1 - \theta)f(y_0) + \theta f(y_1))$$

with parameter  $\theta \in [0, 1]$ .

- The Theta-method is A-stable for  $\theta \geq \frac{1}{2}$ .
- If there exists a constant  $c > 0$  such that  $\theta - \frac{1}{2} \leq ch$ , the method is consistent of second order.
- Show that the Theta-method is well-defined, i.e. that it is possible to calculate  $y_1$ .