

Numerical Analysis of Ordinary Differential Equations

Exercises

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Universität Heidelberg - IWR

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Exercise Sheet 6

Until: Mon, 28.05.2018, Noon (12:00h)

Problem 6.1 (Implicit Runge-Kutta methods)

Consider a 2-stage IRK method of the form

$$\begin{array}{c|cc} u & v & u-v \\ 1-u & 1-u-w & w \\ \hline & 1/2 & 1/2 \end{array}$$

- Show that all these methods are at least of order 2.
- Which conditions have to be satisfied such that they are of order 3?
- Construct a method of order 4.

Problem 6.2 (Implicit Euler)

We aim to solve the IVP

$$u' = -u^3 + 3u^2 + 15u + 2, \quad u(0) = 1$$

using the implicit Euler method. To do this, at each iteration we need to solve a nonlinear equation using Newton's method.

- Write down the Newton's method iteration procedure $F(y_k, y_{k+1}, h_k) = 0$ for the first implicit Euler step of the above IVP.
- How do you choose the initial iterate for Newton's method at each implicit Euler step y_k in practice? Give reasons.
- Find a condition on y_k and the step length h_k that guarantees the convergence of Newton's method in each implicit Euler step.
- Given the initial value $y_0 = 1$, estimate a theoretical upper bound h^* for h_k such that Newton's method will converge for the first implicit Euler step. Use (symbolic) mathematical software for help (e.g. Wolfram Alpha, Maple, Mathematica).
- Investigate the consequences for Newton's method of using an $h_k > h^*$ by plotting $F(y_k, y_{k+1}, h_k)$. Is this advisable in practice? Give reasons.