Numerical Analysis of Ordinary Differential Equations

Exercises

Summer Semester 2018

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Exercise Sheet 7
Until: Mon, 04.06.2018, Noon (12:00h)

Problem 7.1 (Consistency order and stability of LMM)
Consider the explicit multistep method given by

$$y_k + \alpha_1 y_{k-1} + \alpha_0 y_{k-2} = h(\beta_1 f(t_{k-1}, y_{k-1}) + \beta_0 f(t_{k-2}, y_{k-2})).$$

a) Determine α_0, β_1, β_0 depending on α_1 such that the LMM has at least consistency order 2.

b) For which values of α_1 is the method stable?

c) Which method is obtained when $\alpha_0 = 0$ and $\alpha_1 = -1$?

d) Can α_1 be chosen such that the method has convergence order 3?

Problem 7.2 (BDF methods)
Consider the following stiff problem

$$\begin{aligned} u'(t) &= -10u(t) - 100v(t), \\ v'(t) &= 100u(t) - 10v(t), \\ w'(t) &= u(t) + v(t) - tw(t), \end{aligned}$$

which should be solved by a BDF method. To do this effiencently methods of higher order are prefered. Which BDF method introduced in the lecture should be chosen?

Problem 7.3 (**Solvability of BVP**)

Consider for $k \in \mathbb{R}, k \neq 0$ the differential equation

$$u'' = k^2 u, \tag{7.1}$$

with the two linearly independent solutions $e^{k(t-t_0)}$ and $e^{-k(t-t_0)}$.

a) Transform this equation into a first order system for the vector $(u, v)^T$ and determine the fundamental matrix at t for the initial value $\mathbb{I}$ at t_0.

   Note that $\sinh z = \frac{e^z - e^{-z}}{2}$ and $\cosh z = \frac{e^z + e^{-z}}{2}$.

b) On the interval $[a, b]$ consider the boundary value problem that consists of (7.1) and the boundary conditions $u(a) = g_a$ and $u(b) = g_b$. Rewrite this BVP in terms of the first order system with boundary conditions of the form

$$B_a \begin{pmatrix} u(a) \\ v(a) \end{pmatrix} + B_b \begin{pmatrix} u(b) \\ v(b) \end{pmatrix} = \begin{pmatrix} g_a \\ g_b \end{pmatrix},$$

   i.e. determine B_a and B_b.

c) Use Corollary 5.3.13 to determine whether this system has a unique solution for $a = -1$, $b = 1$. If so, determine the solution for $g_a = g_b = 1$.

   Hint: Consider $\cosh(kt)$.