

Numerical Analysis of Ordinary Differential Equations Exercises

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Exercise sheet 3
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Exercise 3.1 (Consistency)

A general explicit Runge-Kutta method with two stages has the following form:

$$y_n = y_{n-1} + h_n F(h_n; t_{n-1}, y_{n-1})$$
$$F(h_n; t_{n-1}, y_{n-1}) = b_1 f_{n-1} + b_2 f(t_{n-1} + h_n c_2, y_{n-1} + a_{21} f_{n-1})$$

Use Taylor-Expansion and verify that both the *modified Euler method* and the *Heun method* are of order 2. Are there any other 2-stage RK methods of order 2?

Exercise 3.2 (Effort and Accuracy)

Consider the IVP

$$u'(t) = -2tu(t)^2, \quad t \geq 1, \quad u(1) = \frac{1}{2},$$

with exact solution $u(t) = \frac{1}{1+t^2}$. Approximate the solution $u(1.4)$ by

- a) four steps of the Euler method
- b) two steps of the Heun method
- c) one step of the classical Runge-Kutta method of order 4.

Compare effort and accuracy.