Numerical Analysis of Ordinary Differential Equations Exercises

Summer semester 2016

Universität Heidelberg - IWR
Prof. Dr. Guido Kanschat, Dr. Dörte Beigel,
Stefan Meggendorfer, Philipp Siehr

Exercise sheet 13
Until: –

Exercise 13.1 (**Questions V**)

a) How is a general boundary value problem defined? And how does the boundary condition simplify, if it is linear?

b) How is the Gâteaux-derivative of a mapping F defined?

c) How is the single shooting method motivated?

d) Describe the variational equation. In which cases could you solve it directly? And what has to be done in the other cases?

e) Where is the variational equation needed to solve BVP? How is the variational equation connected to shooting methods?

f) What is the motivation of the multiple shooting method? How is the method defined?

g) Could a multiple shooting method be used to solve multi-point boundary value problems?

Exercise 13.2 (**Questions VI**)

a) Formulate Newton's method for the problem $f(x) = 0$, $f : \mathbb{R}^n \to \mathbb{R}^m$.

b) Describe the convergence properties of Newton's method.

c) Consider the following BVP: $u''(t) = u(t)$ with $u(0) = u(1) = 1$. How large is the matrix in Newton's method for a multiple shooting method with 5 shooting intervalls?

d) What is the difference between Newton's method and a quasi-Newton method?

e) Describe the gradient method.

f) What is the motiviation for using a Newton's method with *step size control*?

Exercise 13.3 (**Questions VII**)

a) What is an M-Matrix?

b) Formulate the difference quotient for the second derivative. What is the order of that difference quotient?

c) Use the following BVP to explain the finite difference method: $u''(t) = f(t)$ in $[a, b]$ and $u(a) = u(b) = 0$.

d) What are properties of the resulting matrix in the previous question?